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Leap Zagreb Indices Of Tensor Product Of Graphs

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Abstract-The present research paper deals with the study of leap Zagreb indices of tensor product of cycles. Mathematics Subject Classification 2010: 05C12, 05C38, 05C76.

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1. INTRODUCTION

Let $G= (V, E)$ be a simple, connected and finite graph with $|V(G)|=p$ and $|E(G)|=q$. Here the undefined terminologies are referred to [6]. The distance $d(u, v)$ between any two vertices u and v, is the shortest path between them.

Let G_1 and G_2 be the graphs with p vertices and q edges. The tensor product [1] $G \otimes G_2$ of G_1 and G_2 has the vertex set $V(G_1 \otimes G_2) =\{ (u_i v_j) : i=1,2,..., m \text{ and } j=1,2,...,n \}$ and edge set $E(G_1 \otimes G_2)=\{(u_1v_1) (u_2 v_2) : u_1 u_2 \in G_1 \text{ and } v_1 \ v_2 \in G_2.\}$

For a vertex $v \in V(G)$ and a positive integer k, the open k-neighborhood of V in a graph G is denoted by $N_k(V/G)$ and is defined as $N_k(V/G) = \{u \in V(G) : d(u,v)=k \}$. The k-distance degree of a vertex v in G is denoted by $d_k(V/G)$ is defined as the number of k-neighborhoods of the vertex V in G.

$$\text{i.e,, } d_k(V/G) = |N_k(V/G)|$$
$$d_1(V/G) = d(V/G) \text{ for every } v \in V(G).$$

A molecular graph is used to represent a molecule by considering the atoms as vertices of a graph and molecular bonds as edges. A topological index is a numerical value associated with chemical constitution as per the IUPAC terminology. A topological index of a graph is a graph invariant number, calculated from a graph, representing a molecule and applicable in chemistry. The studies on large number of vertex-degree-based graph variants have been made in the current mathematical and mathematico-chemical literature [4]. Among these variants, M_1 and M_2 are called as first and second Zagreb indices respectively. The properties of two Zagreb indices are cited in [2, 3]. The Zagreb indices are introduced by I. Gutman *et al*. [5] in 1972, which way back to 40 years and are defined as

$$M_1(G) = \Sigma\, d_1^{\,2} (v/G)$$
$$M_2(G) = \sum_{uv \in E(G)} d_1 (u/G) d_2 (v/G)$$

Further, a new distance-degree based topological indices based on the second degrees of vertices was introduced by A.M. Naji *et*, al. [7] and are called as leap Zagreb indices i.e., first leap Zagreb index, second leap Zagreb index and third leap Zagreb index of a graph which are defined as

$$LM_1=\sum_{v \in G} d_2^2 (v/G)$$
$$LM_2=\sum_{uv \in E(G)} d_2 (u/G) d_2 (v/G)$$
$$LM_3=\sum_{v \in V(G)} d(v/G) d_2(v/G)$$

Various research works on leap Zagreb indices and tensor product of different graphs have been documented in [7],[8] and [9]. However the study on leap Zagreb indices of tensor product of cycles is scanty. The present research work is a new initiative and motivated by the work of aforementioned authors, we established the results on leap Zagreb indices of tensor product of cycles.

Remark A: A very interesting finding revealed in the tensor product of cycles $C_m \otimes C_n$, for $m, n \geq 3$, is that, the degree of each vertex is 4 attributed to cycle C_m and C_n are both 2-regular graphs that resulted into the product in 4-regular graphs.

2. RESULTS

In this paper analytical methods are applied to obtain results on leap Zagreb indices of tensor product of various cycles.

Theorem 2.1. *Let $C_m \otimes C_n$ be the tensor product of cycle for $m=n=4$.*
Then the leap Zagreb indices are

(i). $LM_1(C_4 \otimes C_4) = 144$
(ii). $LM_2(C_4 \otimes C_4) = 288$
(iii). $LM_3(C_4 \otimes C_4) = 192$

Proof. Consider the tensor product of cycle $C_m \otimes C_n$ for $m=n=4$ with 16 vertices and 32 edges.

(i). By the definition of first leap Zagreb index

$LM_1(C_m \otimes C_n)= \sum_{v \in G} d_2^2 ((u_i\, v_j)/G)$ for $i=1,2,...,m$ and $j=1,2,...,n$.
Since every vertex of $C_4 \otimes C_4$ has 3 vertices which are at distance 2,

then $LM_1(C_4 \otimes C_4) = 16 \times 3^2 = 144$

(ii). By the definition of second leap Zagreb index, $LM_1(C_m \otimes C_n)=$
$\sum_{uv \epsilon E(G)} d_2\ ((u_i\ v_j)G)\ d_2\ ((u_{i+1}v_{j+1})/G)$
for $i=1,2,...,m$ and $j=1,2,...,n$.

Every vertex of each edge in graph $C_4 \otimes C_4$ has 3 vertices with distance 2.

Therefore , $LM_2(C_4 \otimes C_4) = 32 \times 3 \times 3 = 288$.

(iii). By the definition of third leap Zagreb index, $LM_3(C_m \otimes C_n) = \sum_{v\epsilon(G)} d((u_i\ v_j)/G) d_2((u_i\ v_j)/G$, for $i=1,2,...,m$ and $j=1,2,...,n$.

We observe that in $C_4 \otimes C_4$, every vertex has 3 vertices which are at distance 2. From this observation and remark A, we obtain

$$LM_3(C_4 \otimes C_4) = 16 \times 4 \times 3 = 192$$

Theorem 2.2. *Let $C_m \otimes C_n$ be the tensor product of cycle for $m=4$, $n=3$ and $n \geq 5$. Then the leap Zagreb indices are*

(i). $LM_1(C_4 \otimes C_n) = 100n$

(ii). $LM_2(C_4 \otimes C_n) = 200n$

(iii). $LM_3(C_4 \otimes C_n) = 80n$

Proof. Consider the tensor product of cycle $C_m \otimes C_n$ for $m=4$, $n=3$ and $n \geq 5$ with $4n$ vertices and $8n$ edges.

(i). By the definition of first leap Zagreb index

$LM_1(C_m \otimes C_n) = \sum_{v\ \epsilon G} d_2^2\ ((u_i\ v_j)/G)$ for $i=1,2,...,m$ and $j=1,2,...,n$.

Since every vertex of $C_4 \otimes C_n$ with $m=4$, $n=3$ and $n \geq 5$ has 5 vertices which are at distance 2,

then $LM_1(C_4 \otimes C_n) = 4n \times 5^2 = 100n$.

(ii). By the definition of second leap Zagreb index, $LM_1(C_m \otimes C_n)=$
$\sum_{uv \epsilon E(G)} d_2\ ((u_i\ v_j)/G)\ d_2\ ((u_{i+1}v_{j+1})/G)$ for $i=1,2,...,m$ and $j=1,2,...,n$.

Every vertex of each edge in graph $C_4 \otimes C_n$ has 5 vertices with distance 2.

Therefore, $LM_2(C_4 \otimes C_n) = 8n \times 5 \times 5 = 200n$.

(iii). By the definition of third leap Zagreb index, $LM_3(C_m \otimes C_n) = \sum_{v\epsilon(G)} d((u_i\ v_j)/G) d_2((u_i\ v_j)/G$, for $i=1,2,...,m$ and $j=1,2,...,n$.

We observe that in $C_4 \otimes C_n$, every vertex has 5 vertices which are at distance 2. From this observation and remark A, we obtain

$$LM_3(C_4 \otimes C_n) = 4n \times 4 \times 5 = 80n$$

□

Theorem 2.3. *Let $C_m \otimes C_n$ be the tensor product of cycle for $m \geq 3$ (except=4) and $n \geq 3$ (except n=4). Then the leap Zagreb indices are*

(i). $LM_1(C_m \otimes C_n) = 64mn$

(ii). $LM_2(C_m \otimes C_n) = 128mn$

(iii). $LM_3(C_m \otimes C_n) = 32mn$

Proof. Consider the tensor product of cycle $C_m \otimes C_n$ for $m \geq 3$ *(except=4) and* $n \geq 3$ *(except n=4)* with mn vertices and $2mn$ edges.

(i). By the definition of first leap Zagreb index

$LM_1(C_m \otimes C_n) = \sum_{v\ \epsilon G} d_2^2\ ((u_i\ v_j)/G)$ for i=1,2,...,m and j=1,2,...,n.

Since every vertex of $C_m \otimes C_n$ with $m \geq 3$ and $n \geq 3$, has 8 vertices which are at distance 2,

then $LM_1(C_m \otimes C_n) = mn \times 8^2 = 64mn$.

(ii). By the definition of second leap Zagreb index, $LM_1(C_m \otimes C_n)=$
$\sum_{uv \epsilon E(G)} d_2\ ((u_i\ v_j)G)\ d_2\ ((u_{i+1}v_{j+1}/G)$ for i=1,2,...,m and j=1,2,...,n.

Every vertex of each edge in graph $C_m \otimes C_n$ has 8 vertices with distance 2.

Therefore, $LM_2(C_m \otimes C_n) = 2mn \times 8 \times 8 = 128mn$.

(iii). By the definition of third leap Zagreb index, $LM_3(C_m \otimes C_n) = \sum_{v\epsilon(G)} d((u_i\ v_j)/G) d_2((u_i\ v_j)/G)$, for i=1,2,...,m and j=1,2,...,n.

We observe that in $C_m \otimes C_n$, every vertex has 8 vertices which are at distance 2. From this observation and remark A, we obtain

$$LM_3(C_m \otimes C_n) = mn \times 4 \times 8 = 32mn$$

□

Corollary 2.4. *Let $C_m \otimes C_n$ be the tensor product of cycle for $m \geq 3$ (except=4) and $n \geq 3$ (except n=4). From Theorem 2.3, it is observed that the second leap Zagreb index is two times of the first leap Zzagreb index and four times that of third leap Zagreb index.*

Proof : Consider the tensor product of $C_m \otimes C_n$ *for* $m \geq 3$ *(except=4) and* $n \geq 3$ *(except n=4)*. The proof of this result follows from theorem 2.3.

3. CONCLUSION

From the present study, we conclude that the value of second leap Zagreb index for each cycle is two times the value of the first leap Zagreb index which is evidenced in all the theorems.

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